2.0 Current-Carrying Capacity of Busbars
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2.1 Design Philosophy

The current-carrying capacity of a busbar is limited by the maximum acceptable working temperature of the system, taking into account the properties of the conductor material, the materials used for mounting the bars and the limitations of any cables (including their insulation) or devices connected to the bars.

There are two design limits: the maximum permitted temperature rise, as defined by switchgear standards, and the maximum temperature rise consistent with lowest lifetime costs - in the vast majority of cases, the maximum temperature dictated by economic considerations will be rather lower than that permitted by standards.

National and international standards, such as British Standard BS 159 and American Standard ANSI C37.20, give maximum temperature rises as well as maximum ambient temperatures. For example, BS 159:1992 stipulates a maximum temperature rise of 50°C above a 24 hour mean ambient temperature of up to 35°C, and a peak ambient temperature of 40°C. Alternatively, ANSI C37.20 permits a temperature rise of 65°C above a maximum ambient of 40°C, provided that silver-plated (or acceptable alternative) bolted terminations are used. If not, a temperature rise of 30°C is allowed.

These upper temperature limits were chosen to limit the potential for surface oxidation of conductor materials and to reduce the mechanical stress at joints due to cyclic temperature variations. In practice these limitations on temperature rise may be relaxed for copper busbars if suitable insulation materials are used. A nominal rise of 60°C or more above an ambient of 40°C is allowed by EN 60439-1:1994 provided that suitable preconditions are met. A nominal rise of 60°C or more above an ambient of 40°C is allowed by EN 60439-1:1994 provided that suitable preconditions are met. A nominal rise of 60°C or more above an ambient of 40°C is allowed by EN 60439-1:1994 provided that suitable preconditions are met. A nominal rise of 60°C or more above an ambient of 40°C is allowed by EN 60439-1:1994 provided that suitable preconditions are met.

Running busbars at a high working temperature allows the size of the bar to be minimised, saving material and initial cost. However, there are good reasons to design for a lower working temperature.

- A higher maximum temperature implies a wider variation in temperature during the load cycle, which causes varying stress on joints and supports as the bars expand and contract. Eventually, this may lead to poor joint performance and decreased reliability.
- In some cases it is difficult to remove heat from busbar compartments, resulting in ancillary equipment operating in an elevated ambient temperature.
- High temperatures indicate high energy losses. Lowering the temperature by increasing the size of the conductor reduces energy losses and thereby reduces the cost of ownership over the whole lifetime of the installation. If the installation is designed for lowest lifetime cost, the working temperature will be far below the limit set by standards and the system will be much more reliable.

2.2 Calculation of Maximum Current-Carrying Capacity

In engineering terms, the current rating for a busbar depends on the choice of working temperature. The bar is heated by the power dissipated in it by the load current flowing through the resistance, and cooled by radiation to its surroundings and convection from its surfaces. At the working temperature, the heat generation and loss mechanisms, which are highly temperature and shape dependent, are balanced.

2.2.1 Methods of Heat Loss

The heat generated in a busbar can only be dissipated in the following ways:

- Convection
- Radiation
- Conduction

In most cases, convection and radiation heat losses determine the current-carrying capacity of a busbar system. In a simple busbar, conduction plays no part since there is no heat flow along a bar of uniform temperature. Conduction need only be taken into account where a known amount of heat can flow into a heat sink outside the busbar system, or where adjacent parts of the system have differing cooling capacities. Conduction may be important in panel enclosures.
These cooling mechanisms are highly non-linear; doubling the width of a bar does not double the convection loss of a bar. The proportion of heat loss by convection and radiation depends on the conductor size, with the portion attributable to convection normally being greater for a small conductor and less for larger conductors.

### 2.2.1.1 Convection – Natural Air Cooling

The heat dissipated per unit area by convection depends on the shape and size of the conductor and its temperature rise above ambient temperature. This value is usually calculated for still air conditions but can be increased greatly if there is forced air-cooling. Where outdoor busbar systems are concerned, calculations should always be treated as in still (i.e. without wind effect) air unless specific information is given to the contrary.

The following formulae estimate the convection heat loss from a surface in W/m² in free air:

For vertical surfaces:

\[
W_v = \frac{7.66\theta^{1.25}}{L^{0.25}}
\]

For horizontal surfaces:

\[
W_h = \frac{5.92\theta^{1.25}}{L^{0.25}}
\]

For round tubes:

\[
W_c = \frac{7.66\theta^{1.25}}{d^{0.25}}
\]

where:

- \( \theta \) is temperature rise, °C
- \( L \) is height or width of surface, mm
- \( d \) is diameter of tube, mm.

Figure 4 shows the heat loss from a vertical surface \((W_v)\) for various temperature rises plotted against surface height.

![Figure 4](image-url)

**Figure 4 – Heat dissipation by convection from a vertical surface for various temperature rises above ambient**
Figure 5 indicates which formula should be used for various conductor geometries.

Comparing diagrams (a) and (b) and assuming a similar cross-sectional area, it can be seen that the heat loss from arrangement (b) is much larger, provided the gap between the bars is not less than the thickness of each bar.

### 2.2.1.2 Convection Heat Loss – Forced Air Cooling

If the air velocity over the busbar surface is less than 0.5 m/s, the above formulae for $W_v, W_h$ and $W_c$ apply. For higher air velocities the following may be used:

$$W_a = 120 \sqrt{\nu \times A \theta}$$

where:
- $W_a$ is heat lost per unit length from bar, W/m
- $\nu$ is air velocity, m/s
- $A$ is surface area per unit length of bar, m²/m
- $\theta$ is temperature rise, °C.
### 2.2.1.3 Radiation

The rate at which heat is radiated from a body to its surroundings is proportional to the difference between the fourth power of their absolute temperatures and the relative emissivity between the body and its surroundings. Emissivity describes how well a material radiates heat with a perfect radiator (a black body) having a value of unity and a perfectly reflecting surface a value of zero.

Since the amount of radiation depends on the temperature of the bar and its surroundings, bars in enclosed spaces may lose very little heat by radiation. Note that radiation in a particular direction will be influenced by the temperature and condition of any target surface.

The relative emissivity is calculated as follows:

\[
e = \frac{\varepsilon_1 \varepsilon_2}{(\varepsilon_1 + \varepsilon_2) - (\varepsilon_1 \varepsilon_2)}
\]

where:
- \(e\) is relative emissivity
- \(\varepsilon_1\) is absolute emissivity of body 1
- \(\varepsilon_2\) is absolute emissivity of body 2

Typical emissivity values for copper busbars in various surface conditions are:

- Bright metal: 0.10
- Partially oxidised: 0.30
- Heavily oxidised: 0.70
- Dull non-metallic paint: 0.90

The rate of heat loss by radiation from a bar (W/m²) is given by:

\[
W_r = 5.70 \times 10^{-8} \times e(T_1^4 - T_2^4)
\]

where:
- \(e\) is relative emissivity
- \(T_1\) is absolute temperature of body 1, K
- \(T_2\) is absolute temperature of body 2, K (i.e. ambient temperature of the surroundings)

![Figure 6 - Heat dissipation by radiation from a surface assuming relative emissivity of 0.5 and surroundings at 30°C](image-url)
Figure 6 shows the heat loss from a surface against surface temperature. The diagrams in Figure 7 define the effective surface areas for radiation from conductors of common shapes and arrangements. Note that there is no heat loss by radiation from opposing busbar faces since the temperatures are approximately equal.

The ratio of heat dissipated by convection and radiation varies considerably according to the height of the surface and the temperature rise, with radiation becoming less important for smaller bars and lower temperature rises. Figure 8 combines the data from Figure 4 and Figure 6 with the radiation dissipation levels, which are independent of surface height, shown as horizontal bars on the right.

In some countries it is common practice to attempt to increase emissivity to improve the dissipation by radiation by treating the surface of busbars, for example by painting them black. Since the natural emissivity of a copper bar that has been in use for even a short time will be above 0.5 and probably approaching 0.7, the benefit of increasing it to 0.9, although positive, will be small. On the negative side, the paint layer acts as an insulator, reducing the efficiency of the convection process. In general, painting will give little increase and possibly a reduction in the current-carrying capacity of a busbar for a given working temperature. Painting may be worthwhile for very wide bars (where convection is less effective) operating at large temperature rises (where radiation is more effective).
The plots given so far have been in terms of power dissipated per unit area of surface; for engineering purposes it is the heat dissipation per unit length which is of interest. Figure 9 and Figure 10 show the heat dissipation from the major surfaces of single and parallel vertically mounted bars. The relatively small contribution of the horizontal surfaces is ignored in these plots. These plots may be useful in determining a starting point for detailed calculation using the formulae previously given.
2.2.2 Heat Generated by a Conductor

The rate at which heat is generated per unit length of a conductor carrying direct current is the product $I^2R$ watts, where $I$ is the current flowing in the conductor and $R$ is the resistance per unit length. In the case of dc busbar systems, the value for the resistance can be calculated directly from the resistivity of the copper or copper alloy at the expected working temperature. Where an ac busbar system is concerned, the resistance is increased because current density is increased near the outer surface of the conductor and reduced in the middle; this is called skin effect. Eddy currents induced by magnetic fields arising from currents in nearby conductors increase losses further; this is called proximity effect. The calculation of these effects is discussed later; for the present, a correction factor, $S$, is used.

The power dissipated in the conductor is

\[ P = I^2 \times R_0 \times S \]

where:
- $P$ is the power dissipated per unit length
- $I$ is the current in conductor
- $R_0$ is the dc resistance per unit length at the working temperature
- $S$ is the correction factor for shape and proximity.

The process of sizing a busbar is one of iteration. Starting from an arbitrary size and the desired working temperature, the heat power loss from the surface of a one-metre section can be calculated. The electrical power loss for the one-metre section can also be calculated. If the electrical power dissipated is higher than the heat dissipation, the bar is too small; the size should be increased and the calculations repeated until a close match is obtained. Note that the value of resistivity used in the calculation must be corrected for the working temperature and the value of $S$ (the correction for shape and proximity factors) must be recalculated for each size. A very approximate starting point is to assume an average current density of 2 A/mm² in still air and iterate either up or down.
2.2.2.1 Alternating Current Effects – the Factor $S$

The factor $S$, introduced above, is the product of the factors due to skin effect, $S_k$, and proximity factor, $S_p$. Accurate determination of skin and proximity effects is complex and generally requires finite element analysis. In this publication results are presented as curves and, where possible, 'portmanteau formulae' in the form of polynomials or ratios of polynomials valid over a given range of an independent variable such as the scaling factor, $p$, described below. Most of these formulae were found by curve fitting to data computed by finite element analysis. The portmanteau formulae do not always reflect satisfactorily a physical explanation and are usually very inaccurate outside the given range. They are normally accurate to within $\pm 1\%$ within the stated range unless otherwise noted.

The graphs are plotted from the computed data. To make them readable, scaling factors and quasi-logarithmic scales are used.

The resistance/frequency parameter scaling factor combines frequency, resistivity and size. For a busbar configuration of given shape or relative proportions, the ac resistance and inductance may be expressed as a function of a ratio of frequency to dc resistance. Two forms of the scaling factor are used in terms of either

(a) The parameter, $p$, defined as

$$ p = \sqrt{2\mu_0} \sqrt{\frac{f}{R_{dc}}} \approx 1.585 \sqrt{\frac{f}{R_{dc}}} $$

where:

- $f$ is frequency in Hz
- $R_{dc}$ is dc resistance in $\mu\Omega/m$

also

$$ p = \frac{2}{\pi} \frac{\sqrt{A}}{\delta} \approx 0.8 \frac{\sqrt{A}}{\delta} $$

where:

- $A$ is the cross-sectional area
- $\delta$ is the skin depth

The area $A$ and the skin depth must be in comparable units. Thus if $\delta$ is in mm then $A$ must be in mm$^2$.

Sometimes $\sqrt{f}$ is used as the scaling parameter.

The parameter $p$ is chosen since it appears widely in the literature on the subject. The plots for different cross-sectional shapes give clearer curves when using $p$ rather than, say, $p^2$.

For $p \leq 0.5$ the resistance is little different from the dc resistance. $R_{ac}$ is proportional to $p^4$ for low values of $p$ and so proportional to the square of the frequency but, at higher values beyond $p = 5$, the resistance becomes linear with $p$, i.e. proportional to the square root of frequency. If efficiency is a concern, then $p$ will normally be substantially less than 5 and more likely to be in the range 0 to 2. Figure 11 shows how $p$ varies with cross-sectional area at a selection of frequencies for copper at 80$^\circ$C.
In some of the plots the scaling factor may be expressed simply as $\sqrt{\frac{I}{R_{dc}}}$ or in terms of $\gamma$.

(b) The ratio $\gamma$, defined as

$$\gamma = \frac{Leading\ dimension}{\delta,\ skin\ depth}$$

This can often give a physical idea of the effect of size of a conductor as, for example, for round bars where $\gamma = \frac{d}{\delta}$ and $d$ is the bar diameter. For low losses $\gamma \leq 1$ is desirable.

For a given busbar geometry, the values of $\gamma$ and $p$ are proportional to each other.

**Quasi logarithmic scales:** The resistance ratio or shape factor, $S = R_{ac}/R_{dc}$, approaches unity at low frequencies. In some graphs the logarithm of $(S-1)$ is plotted against the independent variable such as scaling parameter, $p$. This method shows the approach to unity in more detail. However, the values on the $S$ scale are labelled as the actual values of $S$ and not of $(S-1)$. Such plots typically go down to $S = 1.01$ as covering a useful range.

**Busbar configuration:** The choice of busbar configuration will depend on many factors. Round bars, tubes, strips or channels are common. Factors to be taken into consideration include frequency and the presence of harmonics, cost of materials, degree of compactness sought, cooling arrangements, magnetic mechanical forces and cost of support structures, inductance and, for longer bars, capacitance.

From Figure 11 it will be seen that, for $p < 2$, the cross-sectional area of bars is limited to 400 mm$^2$. At a typical current of 2 A mm$^{-2}$ this is a current of 800 A. The effective ac resistance may be reduced by:

1. Using two bars per phase separated by a suitable distance along their length. This possibility is discussed for some shapes in later sections.
2. Using multiple bars for a phase and transposing them along the length of the bar.
3. Interleaving phases but then the design becomes more detailed.
2.2.2.1.1 Skin Effect and Skin Depth

Current flowing in the inner parts of a conductor produces a magnetic field inside the conductor that circulates around the axis of the conductor. This alternating magnetic field induces an electric field that drives currents in such a way that the current in the centre of the conductor is reduced while the current in the outer parts is increased. These induced recirculating currents are referred to as eddy currents. The overall result is that the current density is highest near the surface and falls off towards the interior.

A measure of the extent of the fall off (and so of the effect of the ac nature) is the skin depth, sometimes called the depth of penetration, and usually given the symbol $\delta$.

$$\delta = \frac{\rho}{\pi \mu_0 f} = \frac{2\rho}{\mu_0 \omega}$$

where:
- $\rho$ is resistivity of conductor material, $\Omega \ m$
- $\mu_0$ is permeability of free space, $4\pi \times 10^{-7} \ H/m$
- $f$ is frequency, $Hz$
- $\delta$ is skin depth, $m$

Or, in alternative units,

$$\delta = \frac{50}{\pi} \sqrt{\frac{\rho}{f}} \approx 15.915 \sqrt{\frac{\rho}{f}}$$

where:
- $\rho$ is resistivity of conductor material, $n\Omega \ m$
- $f$ is frequency, $Hz$
- $\delta$ is skin depth, $mm$

Hence, for copper at 20°C ($\rho = 17 \ n\Omega \ m$),

$$\delta = \frac{65.6}{\sqrt{f}}$$

$\approx 9.27 \ mm$ at 50Hz
$\approx 8.46 \ mm$ at 60 Hz
$\approx 3.28 \ mm$ at 400 Hz

About 85% of the current is carried in a layer of thickness equal to the skin depth in conductors that are much thicker than the skin depth. In such thick conductors the resistance is as if the current flowed with uniform current density in a layer of thickness equal to the skin depth.

Figure 12 shows how the resistivity of a typical grade of copper varies with temperature and Figure 13 shows how the skin depth varies with temperature at typical power frequencies.
Figure 12 - Resistivity of typical HC copper (101.5% IACS) as a function of temperature. Note $10\,\Omega\,\text{m} = 1\,\mu\Omega\,\text{cm}$

Figure 13 - Skin depth of typical HC copper (101.5% IACS) at 50 Hz, 60 Hz and 400 Hz as a function of temperature
If the dimensions of the bar are larger than about half the skin depth, then ac effects will be important. In such a case, the extra thickness may increase the losses rather than reduce them, but only slightly.

A useful measure of the ac effect in determining the ac resistance of a bar is the **Shape Factor**, \( S \), defined as

\[
S = \frac{R_{ac}}{R_{dc}}
\]

where:

- \( R_{dc} \) is the dc resistance
- \( R_{ac} \) is the ac resistance

thus,

\[
R_{dc} = \frac{\rho}{A}
\]

where:

- \( R_{dc} \) is resistance, \( \Omega/m \)
- \( A \) is cross-section, \( m^2 \)
- \( \rho \) is resistivity, \( \Omega \cdot m \)

or,

\[
R_{dc} = \frac{1000\rho}{A}
\]

where:

- \( R_{dc} \) is resistance, \( \mu\Omega/m \) or \( m\Omega/km \)
- \( A \) is cross-section, \( mm^2 \)
- \( \rho \) is resistivity, \( n\Omega \cdot m \)

and

\[
R_{ac} = S \cdot R_{dc}
\]

Although computing the bar resistance is straightforward, Figure 14 is given either as a check or for approximate estimation.

Figure 14 – dc resistance of typical HC Copper (101.5% IACS) versus area at 20°C and 80°C

One would hope in any design that the shape factor, \( S \), would be not much more than unity.

The determination of the shape factor is a principal feature of this section.
The power, $P$, dissipated as heat in a bar carrying current, $I$, is then

$$P = S R_{dc} I^2$$

or

$$P = S \frac{\rho}{A} I^2$$

where:
- $P$ is the power dissipated per metre length, W/m
- $R_{dc}$ is the resistance per metre length, $\Omega$/m
- $S$ is shape factor
- $\rho$ is resistivity, $\Omega$ m
- $A$ is cross-sectional area, m$^2$

The precise calculation of eddy current in conductors of general shape requires finite difference or other numerical computational procedures. These procedures require extensive programme writing or access to one of the commercially available programmes and practice in its use. However, for some common configurations there are exact (if complicated) formulae, while for others there are simpler approximations and graphical methods. These are given in '2A Shape and Proximity Factors for Typical Configurations'.

2.2.2.1.2 Proximity Factor, $S_p$

With conductors in parallel there are also eddy current mechanisms and further current redistribution in a conductor caused by magnetic fields arising from currents in neighbouring conductors. This is the proximity effect.

In the case of a single bar, the eddy current mechanism tends to drive current to flow in outer regions of the bar.

In the case of adjacent conductors carrying anti-parallel currents, the effect is to draw current flow towards the facing surfaces of the bars. For round bars or tubes which are carrying anti-parallel currents there is an increase in the current density at those parts of the bar that face each other, so losses are higher. For closely spaced parallel strips with their long sides facing each other, there is still a concentration of current near the ends of the strips, but overall the current is more uniformly distributed across the width and overall losses can be reduced.

The proximity factor is sometimes expressed as $S_p$ so that the effective resistance, $R_{ac}$, as given by:

$$\frac{R_{ac}}{R_{dc}} = S S_p$$

In section 2A, which gives charts and equations for common busbar shapes and arrangements, the shape and proximity factors are given either separately as $S$ and $S_p$ or combined together as $R_{ac}/R_{dc}$.

2.3 Conclusion

The design of a busbar system for a particular duty is quite a complex issue. It requires an iterative approach to arrive at a size and disposition that delivers the energy efficiency levels appropriate to the duty factor of the application while ensuring good reliability and safety.
2A Shape and Proximity Factors for Typical Configurations

2A.1 Skin and Proximity Factors for Common Busbar Shapes

2A.1.1 Single Solid Rods

2A.1.1.1 Shape Factor for Single Solid Rods

For a rod of radius $a$ calculate $\gamma$:

$$\gamma = \frac{a}{\delta}$$

where $\delta$ is skin depth.

Substituting for $\delta$,

$$\gamma = 2\sqrt{\frac{f}{\pi R_{dc}}} \approx 1.121 \sqrt{\frac{f}{R_{dc}}}$$

where:

- $f$ is frequency, Hz
- $R_{dc}$ is resistance in $\mu\Omega/m$

then,

$$\gamma = \frac{1}{\sqrt{2}} p = 0.707 p$$

$S_p$ can be calculated from the following approximate formulae:

for $\gamma < 1.98$:

$$S_p \approx 1 + \frac{5\gamma^4}{240 + 4\gamma^4}$$

Error is <0.02% if $\gamma < 1.68$; <0.1% if $\gamma < 1.98$

for $\gamma > 3.9$:

$$S_p \approx \frac{2\gamma + 1}{2} + \frac{3}{32\gamma}$$

Error is <0.8% if $\gamma > 3.9$, <0.2% if $\gamma > 5$

In the intermediate range $1.6 < \gamma < 3.9$ the following formula is within 0.2%:

$$S_p \approx 0.84 - 0.13\gamma + 0.22\gamma^2 - 0.0246\gamma^3$$

Errors lie between -0.2% and +0.15%.
Computations using the formulae may be checked by comparison with the graphs in Figure 15. These graphs might also be used directly for approximate working.

Figure 15 – Plots of shape factor versus $\gamma$. In the upper left-hand figure, for low values of $\gamma$, the dotted line shows the exact formula and the solid line the given approximation. The difference is scarcely discernable.

2A.1.1.2 Proximity Factor for Single Solid Rods

For the round bar calculate $\eta$,

$$\eta = \frac{s}{2a}$$

where:
- $s$ is the spacing of the centres
- $a$ is the bar radius.
Then the proximity factor, for \( \eta > 2 \)

\[
S_p = \frac{1}{\sqrt{1 - \frac{A}{\eta^2}}}
\]

\( \eta > 2 \) means that the minimum space between the opposing curved faces is greater than the bar diameter.

The factor \( A \) in the above equation is a function of \( \gamma = \frac{a}{\delta} \) and approximately for \( \gamma < 2 \)

\[
A = \frac{\gamma^4(1 - 0.128\gamma^2)}{5.705 + 1.88\gamma^2}
\]

This relationship is plotted in Figure 16.

**Figure 16** - Factor \( A \) for round bars as a function of \( \gamma \)
For three phase systems calculate $\mu$:

$$\mu = \frac{A(\gamma)}{\eta^2}$$

where:

$$\eta = \frac{s}{2a}$$

Then for a triangular arrangement of three phase busbars,

$$S_p_{\text{triangular}} \approx S_p \left(1 + \frac{\mu}{4} - \frac{5\mu^2}{24} - \frac{3\mu^3}{8}\right)$$

And for a flat arrangement the average proximity factor is

$$S_p_{\text{flat}} \approx S_p \left(1 + \frac{\mu}{4} - \frac{\mu^2}{8} + \frac{5\mu^3}{24}\right)$$

The factors are plotted in Figure 18 and Figure 19.
Figure 18 – Mean proximity factors for flat arrangement of round bars carrying balanced three phase currents

Each curve is for the given value of

\[ \gamma = \frac{\text{bar radius, } a}{\text{skin depth, } \delta} \]
Figure 19 - Mean proximity factors for delta arrangement of round bars carrying balanced three phase currents
2A.1.2 Single Tubes

2A.1.2.1 Shape Factors for Single Tubes

Calculate parameters $g$ and $\beta$ as:
\[ g = \frac{t}{\delta} \]

and
\[ \beta = \frac{t}{a} \]

where:
- $a$ is radius
- $\delta$ is skin depth
- $t$ is wall thickness

The units for $a$, $\delta$, and $t$ must be the same.

The approximate formula for $S$ is
\[ S = 1 + A(g) \left\{ 1 - \frac{\beta}{2} - \beta^2 B(g) \right\} \]

where $A$ and $B$ are given by:

for $g < 1.6$:
\[ A(g) = \frac{28g^4}{315 + 12g^4} \]
\[ B(g) = \frac{56}{211 + 4g^4} \]

for $1.59 < g < 3.9$:
\[ A(g) = -1.22 + 1.07g \]
\[ B(g) = \frac{1827 - 300g}{3005 + 1555g} \]

for $g > 3.9$:
\[ A(g) = g - 1 \]
\[ B(g) = \frac{3}{8g - 5} \]

Figures 20 to 24 show plots of shape factor for various tube sizes.
Shape factor, $S = \frac{R_{ac}}{R_{dc}}$

$\beta = \frac{\text{Wall thickness, } t}{\text{Tube outer radius, } a}$

Figure 20 - Shape factor for tubes

$g = \frac{\text{Wall thickness, } t}{\text{Skin depth, } \delta}$

Figure 21 - Shape factor for tubes with low values of shape factor
Figure 22 – Shape factor for tubes

Wall thickness, \( t \)
Tube outer radius, \( a \)

\[ S = \frac{R_{ac}}{R_{dc}} \]

\[
\sqrt{\frac{f}{R_{dc}, \mu\Omega/m}}
\]

Figure 23 – The shape factor computed from the Bessel function formula using \( \sqrt{f R_{dc}} \) as the frequency parameter
2A.1.2.2 Proximity Factors for Tubes

As for the round bar calculate $\eta$,

$$\eta = \frac{s}{2a}$$

where:
- $s$ is the spacing of the centres
- $a$ is the bar radius.

For moderate to large spacings, i.e., for $\eta > 2$,

$$S_p = \frac{1}{\sqrt{1 - \frac{A}{\eta^2}}}$$

with the factor $A$ now given by

$$A = a(x) + p(x) \left[ 1 - \gamma_\lambda^2 - \frac{(1 - \gamma_\lambda) x^2}{400 + x^2} \right]$$

where:
- $x = \sqrt{2} \gamma_\lambda$
- $\gamma_\lambda = \sqrt{b(2 - \beta)}$
- $\gamma = \frac{a}{\delta}$
- $a(x) = \frac{x^4 (1 - 0.064 x^2)}{22.8 + 3.76 x^2}$
- $p(x) = \frac{x^4 (1 - 0.134 x^2)}{25 + 3.973 x^2}$
NOTE: The factor $a(x)$ is not the same as the radius $a$. ‘$a(x)$’ is a term normally adopted in the literature and $a$ and $a(x)$ are unlikely to be confused once the difference is established.

For the single phase case we have Figure 25 which plots the factor $A$ against $\beta = t/a$. This factor $A$ is then used in Figure 26 to find $S_p$, the proximity factor for values of $\eta = s/2a = s/d$.

![Figure 25 - Factor $A$ versus $\beta = t/a$ for values of $g = t/\delta$](image)

![Figure 26 - Proximity factor for single phase tubes as function of the factor $A$ for values of $\eta = s/2a = s/d$](image)

These graphs may be combined into a joint chart as in Figure 27.
Figure 27 - Proximity factor for single phase tubes. \( \beta = t/a \), \( g = t/\delta \) and \( \eta = s/2a = s/d \) with \( t = \) wall thickness, \( a = \) radius, \( d = \) diameter, \( \delta = \) skin depth.

The chart is used by starting with the value of \( \beta \) on the vertical axis of the lower plot, drawing a horizontal line to the curve for the value of \( g \) to obtain the value of the factor \( A \). Next, draw a line vertically to the upper plot to intersect the line for the value of \( \eta \). Finally move horizontally to the left to read of the proximity coefficient, \( S_p \), from the vertical axis of the upper plot. In the illustrated case, \( \beta = 0.45, g = 0.8, A = 0.42, \eta = 3 \) and \( S_p = 1.025 \).
For low values of the parameter $A$, and when $g$ is small, the charts just described are not very accurate. Thus Figure 28 and Figure 29 may be used in these circumstances.

![Graph showing factor $A$ for proximity loss factor as a function of $g = t/\delta$ for various values of $\beta = t/a$]

The formula just derived applies to a single phase system with two parallel conductors. For three phase systems calculate $\mu$:

$$\mu = \frac{A}{\eta^2}$$

where:

$$\eta = \frac{s}{2a}$$

Then for a triangular arrangement of three phase busbars

$$S_{\text{triangular}} \approx S_p \left( 1 + \frac{\mu}{4} - \frac{5\mu^2}{24} - \frac{3\mu^3}{8} \right)$$

And for a flat arrangement the average proximity factor is

$$S_{\text{flat}} \approx S_p \left( 1 + \frac{\mu}{4} - \frac{5\mu^2}{8} + \frac{5\mu^3}{24} \right)$$
For small spacings, i.e. for $\eta < 2$, the approximate formulae are rather bulky but the chart given in Figure 29 can be used.

The parameter $x$ is given by:

$$x = \sqrt{2} \frac{a}{\delta} \sqrt{\beta(2 - \beta)}$$

where:
- $\delta$ is skin depth
- $a$ is tube radius

and

$$\beta = \frac{t}{a}$$

where:
- $t$ is the tube wall thickness.

Also, in SI units,

$$x = \frac{2\pi \mu_0 f}{R_{dc}}$$

with
- $R_{dc}$ in $\mu\Omega/m$ and
- $f$ in Hz

$$x = \frac{1}{10} \sqrt{\frac{f}{R_{dc}}} \approx 0.89 \sqrt{\frac{f}{R_{dc}}} \approx \frac{1}{1.125} \sqrt{\frac{f}{R_{dc}}}$$
2A.1.3 Single Rectangular Sections

We consider here first single bars of rectangular cross-section which might be called strips or even straps. In subsections we consider single phases divided into two strips, separated from each other and carrying currents in the same direction. This leads to reduced losses. Then we consider a pair of bars carrying anti-parallel currents to form a single phase circuit. Finally, we consider three phase arrangements of bars carrying balanced positive phase sequence currents.

A portmanteau formula is given for a single bar, but not for the other arrangements, since none are available. The reason for the dearth is partly because it requires two or more parameters to describe the shape. Thus, resort must be made to graphs or detailed computation using a fairly complicated programme.

2A.1.3.1 Shape Factor for Rectangular Bars

There are no analytic formulae for this case. Resort must be made to numerical methods such as finite elements, finite differences or current simulation, i.e. particle elements.

Recollect that the parameter $p$ is defined as:

$$p = \sqrt{2} \mu_0 \sqrt{\frac{f}{R_{dc}}} \approx 1.585 \sqrt{\frac{f}{R_{dc}}}$$

where:

- $f$ is frequency in Hz
- $R_{dc}$ is dc resistivity in $\mu\Omega/m$

and

$$p = \sqrt{\frac{2}{\pi}} \sqrt{\frac{A}{\delta}} \approx 0.8 \frac{\sqrt{A}}{\delta}$$

where:

- $A$ is the cross-sectional area
- $\delta$ is the skin depth

The area $A$ and the skin depth must be in comparable units. Thus if $\delta$ is in mm then $A$ must be in mm$^2$.

The following plots are of shape factor for strips of different cross-sectional proportions.
Figure 30 - \( \frac{R_{ac}}{R_{dc}} \) as a function of the parameter \( \sqrt{f/R_{dc}} \).

Figure 31 - \( \frac{R_{ac}}{R_{dc}} \) as a function of the parameter \( \sqrt{f/R_{dc}} \). The range of the parameter is narrower than in the previous figure.
At low values of \( \rho \) the curves for the different aspects ratios cross, as illustrated in Figure 32.

![Figure 32 - Shape factor, \( R_{ac}/R_{dc} \), as a function of the parameter \( \sqrt{f/R_{dc}} \). Note the logarithmic scale for the shape factor and that the lines for constant \( b/a \) cross.](image)

Figure 32 - Shape factor, \( R_{ac}/R_{dc} \), as a function of the parameter \( \sqrt{f/R_{dc}} \). Note the logarithmic scale for the shape factor and that the lines for constant \( b/a \) cross.

For values of \( \sqrt{f/R_{dc}} < 1.25 \) the thin sectioned bar has a slightly higher shape factor than the square conductor, whereas for higher values a thin shape gives a lower shape factor. For copper at 20°C with \( \rho = 17 \, \text{n}\Omega \, \text{m} \) the transitional cross-sectional area is about 500 mm\(^2\); i.e. in old units, about 1 sq in. One may, however, prefer a thin strip of say 6 mm by 80 mm, which is more readily cooled because it has a larger vertical flat face. The increase in losses is of the order of one percent.

![Figure 33 - Shape factor versus \( 2b/\delta \) for various values of \( b/a \)](image)

Figure 33 - Shape factor versus \( 2b/\delta \) for various values of \( b/a \)
2A.1.3.2 Approximate Portmanteau Formula for Single Bars

An approximate formula which was made by curve fitting is:

\[ S = \frac{R_{ac}}{R_{dc}} = 1 + \frac{a_s f^4}{1 + b_s p^2 + c_s p^4} \]

with

\[ f \text{ in Hz and } R_{dc} \text{ in } \mu \Omega/m \]

\[ a_s = \frac{1 - 0.2944A + 0.1795A^2}{147.8 - 31.55A + 11.57A^2} \]

\[ b_s = \frac{1 - 0.9334A + 0.91A^2}{31.81 - 6.77A + 2.46A^2} \]

\[ c_s = \frac{1 - 0.41A + 0.11A^2}{226.46 - 5.3A + 5.35A^2} \]

with \( A = \frac{b}{a} \)

2A.1.4 Parallel Bars

2A.1.4.1 Proximity Factor - Anti-parallel Currents

The currents in adjacent bars induce eddy currents in neighbours. For thin wide strips placed with their long sides facing each other, there is usually a reduction in losses as the current becomes distributed more uniformly across the width of the strip. For lower values of \( \frac{b}{a} \) the losses increase. This is illustrated in the next graphs.

If the strips are too close together, the flow of coolant between them may be impaired, the temperature may rise, the resistance increase and so the ohmic losses become larger.

Figure 34 (a–e) shows the shape factor, \( \frac{R_{ac}}{R_{dc}} \), as a function of the spacing expressed as \( \frac{s}{a} \) for various values of \( \sqrt{f/R_{dc}} \). As before, \( f \) is in Hz and \( R_{dc} \) in \( \mu \Omega/m \).
Shape Factor, $S = \frac{R_{ac}}{R_{dc}}\sqrt{\frac{f}{R_{dc}}}$

(b) $b/a=2$

(c) $b/a=4$

(d) $b/a=8$
2A.1.4.2 Proximity Factor – Parallel Currents

If a single busbar is replaced by two busbars of the same total cross-section, the losses may be reduced for one or more of three reasons:

1. The exposed surface area will be larger, the cooling more effective, the temperature lower, the resistivity lower and so losses lower.
2. The shape factor $R_{ac}/R_{dc}$ will be reduced since the ratio for each bar will be lower. The reason for this is that the factor $\sqrt{f/R_{dc}}$ will be less for each bar than for the single bar, since $R_{dc}$ of each bar will be twice that of a single bar. A lower value of $\sqrt{f/R_{dc}}$ means a lower shape factor.
3. The two new bars may be of improved shape so as to give rise to an even lower shape factor than the original design of single bar.

In considering how to design a divided bar, one may perhaps think of an original single bar and split it into two equal halves – for this reason, the width of the bars in this section is $a$ rather than $2a$, as used elsewhere. This is the idea behind the next plots.
Shape factor, \( S = \frac{R_{ac}}{R_{dc}} \sqrt{\left( \frac{b}{a} \right)} \)

- (b) \( b/a = 2 \)
- (c) \( b/a = 4 \)
- (d) \( b/a = 8 \)
2A.1.5 Three-phase Configurations

In the following plots, the resistance to balanced three-phase positive sequence currents is shown. There will also be zero sequence and negative sequence voltages but these produce no losses and are anyway smaller than the positive phase sequence voltages.

2A.1.5.1 Linear Plots

The shape factor $R_{ac}/R_{dc}$ is shown in Figure 36 as a function of the parameter $p$ for various aspect ratios of the bar layout.
Parameter, $\alpha/a$

- **(b) $\alpha/a = 8$**
  - $R_{ac}/R_{dc} = \sqrt{4ab}$
  - $s = \sqrt{4ab}$

- **(c) $\alpha/a = 4$**
  - $R_{ac}/R_{dc} = \sqrt{4ab}$
  - $s = \sqrt{4ab}$

- **(d) $\alpha/a = 2$**
  - $R_{ac}/R_{dc} = \sqrt{4ab}$
  - $s = \sqrt{4ab}$
Figure 36 – Linear plots of $R_{ac}/R_{dc}$ versus $p$ where $p = 1.5853 \sqrt{\frac{f}{R_{dc}}}$ with $R_{dc}$ in $\mu\Omega/m$ and $f$ in Hz.
2A.1.5.2 Logarithmic Plots

For lower losses, using a pseudo-logarithmic scale for $R_{ac}/R_{dc}$ gives clearer plots, as shown in Figure 37.

![Graphs showing logarithmic plots for different parameter values](image)

- (a) $a/b = 1/16$
  - Parameter, $p$
  - $a/b = 1/8$
  - Parameter, $p$
  - $a/b = 1/4$
  - Parameter, $p$
Figure 37 – Quasi logarithmic plots of $R_{ac}/R_{dc}$ versus $p$ where $p = 1.5853 \sqrt{\frac{f}{R_{dc}}}$ with $R_{dc}$ in $\mu\Omega/m$ and $f$ in Hz